GCE

# Mathematics 

## Advanced GCE

Unit 4730: Mechanics 3

## Mark Scheme for January 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## Annotations and abbreviations

| Annotation in scoris | Meaning |
| :--- | :--- |
| $\checkmark$ and $\boldsymbol{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | lgnore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| $\hat{\wedge}$ | Omission sign |
| MR | Misread |
| Highlighting |  |
| Other abbreviations | Meaning |
| in mark scheme | Mark for explaining |
| E1 | Mark for correct units |
| U1 | Mark for a correct feature on a graph |
| G1 | Method mark dependent on a previous mark, indicated by * |
| M1 dep* | Correct answer only |
| cao | Or equivalent |
| oe | Rounded or truncated |
| rot | Seen or implied |
| soi | Without wrong working |
| www |  |

## Subject-specific Marking Instructions for GCE Mathematics Pure strand

Annotations should be used whenever appropriate during your marking
The $A, M$ and $B$ annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

C
The following types of marks are available.
M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

## A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore MO A1 cannot ever be awarded.

## B

Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the $M$ marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, $A$ and $B$ marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
$f \quad$ Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

## When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

## When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.
ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for $g$. E marks will be lost except when results agree to the accuracy required in the question.

For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.
'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | (i) | Triangle of velocities/momentum <br> All correct <br> Use of Pythagoras' theorem to find $I$ $I=0.075$ | M1 <br> A1 <br> M1 <br> A1 <br> [4] | For right angled triangle with at least one side correctly shown ( $2.5,2,20 I$ or $0.125,0.1, I$ ) or vector equation $\left(v_{1}, v_{2}\right)=$ $(0,20 I)+(2,0)$ with at least 3 of the 4 components on the RHS correct $400 I^{2}+2^{2}=2.5^{2} \text { or } I^{2}=0.125^{2}-0.1^{2}$ | may be implied by $v_{1}^{2}+v_{2}{ }^{2}=$ $2.5^{2}$ or $\sin \alpha=0.6$ |
| 1 | (ii) | Components of velocity parallel to the wall before and after are 2 and 2 <br> Components of velocity perpendicular to the wall before and after are (-) 1.5 and $1.5 e$ $\left[2^{2}+(1.5 e)^{2}=5\right]$ <br> Coefficient is $\frac{2}{3}$ or 0.667 | B1 <br> B1 <br> M1 <br> A1 <br> [4] | For using $v_{1}^{2}+v_{2}^{2}=5$ Must be perp to wall | may be implied |
| 2 | (i) | $\begin{aligned} & 2 m u \cos \alpha-m u \cos \alpha=2 m a+m b \\ & 0.5(u \cos \alpha+u \cos \alpha)=b-a \end{aligned}$ <br> Comp of B's velocity along l.o.c. is $u \cos \alpha$ Establishing B's speed unchanged | M1 <br> M1 <br> A1 <br> A1ft <br> A1 <br> [5] | For using the p.c.m. parallel to l.o.c. <br> For using NEL parallel to l.o.c. <br> for both p.c.m and NEL correct \& consistent dep on M1M1 gained by stating vel perp l.o.c. still $u \sin \alpha$, hence result, dep on all previous marks | allow sign errors, $m / 2 m$, sin/cos allow sign errors, e left in <br> or by showing speed is still $u$ condone 'vertical' in this part |
| 2 | (ii) | $a=0$ <br> correct interpretation of direction of $A$ <br> Direction of $B$ is at angle $\alpha$ to l.o.c.., with an indication that removes ambiguity (eg in sketch) | B1 <br> B1 <br> B1 <br> [3] | may be shown in (i) perp to l.o.c. | condone 'vertical' for perpendicular, accept sketch, and refs to sketch in (i) |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | (i) | $\begin{aligned} & 0.3 v(\mathrm{~d} v / \mathrm{d} x)=-1.2 v^{3} \\ & {\left[-v^{-1}=-4 x+A\right]} \\ & {\left[-u^{-1}=0+A\right]} \\ & v=\frac{u}{4 u x+1} \end{aligned}$ | M1 <br> A1 <br> M1* <br> *M1 <br> A1 <br> [5] | For using Newton's second law and $a=v(\mathrm{~d} v / \mathrm{d} x)$ <br> For finding $\mathrm{d} v / \mathrm{d} x$ in terms of $v$ and attempting to integrate <br> For using $v(0)=u$ <br> AG | allow missed - sign / stray $g$ / missed 0.3 <br> allow $A / v=B x+C$ oe |
| 3 | (ii) | $\begin{aligned} & \int(4 u x+1) d x=\int u d t \\ & 2 u x^{2}+x=u t+B \\ & {[(2 \times 4-9) u=-2]} \\ & u=2 \end{aligned}$ | M1* <br> A1 <br> *M1 <br> A1 <br> [4] | For using $v=\mathrm{d} x / \mathrm{d} t$, separating the variables and attempting to integrate one side <br> For using $x(0)=0$ (may be implied by absence of $B$ ) and $x(9)=2$ - dep on int being done | $-1.2 v^{3}=0.3 \mathrm{~d} v / \mathrm{d} t$ and attempt to int one side M1* $8 t=1 / v^{2}-1 / u^{2}$ and subst for $v$ A1 then as main scheme |
| 4 | (i) | $\begin{aligned} & \text { EE gain }=44.1 x^{2} \div(2 x 0.75) \\ & \text { PE loss }=1.8 g(0.75+x) \\ & {\left[x^{2}-0.6 x-0.45=0\right]} \\ & \\ & \text { Extension is } 1.03 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \\ & \text { M1 } \\ & \\ & \text { A1 } \\ & {[4]} \end{aligned}$ | ignore signs <br> For using EE gain = PE loss | allow use of $(e+x)$ for $x$ <br> $44.1 x^{2}-26.46 x-19.845=0$ allow sign errors 1.0348469... |
| 4 | (ii) | $\frac{44.1 \times 1.03}{0.75}-1.8 \times 9.8=-1.8 \ddot{x}$ <br> Acceleration is $-24.0 \mathrm{~ms}^{-2}$ | M1 <br> M1 <br> A1ft <br> A1 <br> [4] | For using $T=\lambda x / L$ For using Newton's $2^{\text {nd }}$ law <br> ft their '1.03' from (i) direction must be clear | allow missed $g, m$, sign error <br> allow sign error $\begin{aligned} & 1.03 \rightarrow-23.84666 \\ & 1.035 \rightarrow-24.01 \\ & \hline \end{aligned}$ |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | (i) | $84.5 \times 12 L / 13=T(2 L)$ <br> Tension is 39 N | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & {[3]} \end{aligned}$ | For taking moments about $B$ for $B C$ must use $12 / 13$ for $\cos \beta$ | must be 2 terms involving $T$, $L$, 84.5 and $\sin / \cos \beta$ |
| 5 | (ii) | $\begin{aligned} & X=39 \times 5 / 13 \\ & Y=84.5-39 \times 12 / 13 \end{aligned}$ <br> $X$ is to the left and $Y$ is upwards | M1 <br> A1 FT <br> A1 FT <br> A1cao <br> [4] | For resolving forces on $B C$ horiz or vert explicit expression for $X$ explicit expression for $Y$ AG (numerical values - must be correct) dep M1A1A1 | must involve their $T$ and $\sin /$ cos $\beta$ accept on diagram |
| 5 | (iii) | $\begin{aligned} & 84.5 \times L \cos \alpha+48.5 \times 2 L \cos \alpha=15 \times 2 L \sin \alpha \\ & {\left[\tan \alpha=\frac{84.5+97}{30}\right]} \\ & \alpha=1.41^{\mathrm{c}} \text { or } 80.6^{\circ} \end{aligned}$ | $\begin{gathered} \text { M1* } \\ \text { A1 } \\ \text { *M1 } \\ \text { A1 } \\ \text { [4] } \end{gathered}$ | For taking moments about $A$ for $A B$ <br> For obtaining a numerical expression for $\tan \alpha$ | must involve 3 terms, 84.5, 48.5, $15, \sin \alpha$ and $\cos \alpha$; allow sign errors, L/2L similar scheme for those who take moments about $A$ for whole system |
| 6 | (i) | $\begin{aligned} & {[0.4 \pi=2 \pi / n]} \\ & n=5 \end{aligned}$ <br> Distance $O A$ is 0.8 m | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \\ & {[4]} \end{aligned}$ | For using $T=2 \pi / n$ <br> For using $v_{\text {max }}=n(O A)$ |  |
| 6 | (ii) | $\begin{array}{\|l} \hline[x=0.8 \cos (5 \times 1)] \\ x=0.227 \\ {[\dot{x}=-0.8 \times 5 \sin (5 \times 1)]} \\ \text { Velocity is } 3.84 \mathrm{~ms}^{-1} \end{array}$ | M1 <br> A1 <br> M1 <br> A1 <br> [4] | For using $x=a \cos n t$ <br> For using $\dot{x}=-a n \sin n t$ | Use of $v^{2}=n^{2}\left(a^{2}-x^{2}\right)$ M1 <br> Direc needs to be shown for A1 |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | (iii) | $t$ and $x$ for one point $t$ and $x$ for second point $t$ and $x$ for third point correctly stating precisely 3 points <br> If B1 or B0 scored (out of first 4) on above scheme, allow, subject to max mark 2, Number of occasions is 3 | $\begin{gathered} \text { B2 } \\ \text { B1 } \\ \text { B1 } \\ \text { B1 } \\ \\ \\ \text { (M1) } \\ \text { (A1) } \\ \hline[5] \\ \hline \end{gathered}$ | Values of $t$ are $=0.257,0.372,0.885$ <br> Values of $x$ are $0.227,-0.227,-0.227$ <br> sc all $3 x$ values B2 <br> all $3 t$ values B2 <br> one $t$ value B1 <br> one $x$ value B1 <br> For $t=1 \approx 0.8 T \rightarrow 3 / 4 T<1<4 / 4 T$ or equiv | $0.4 \pi-1,1-0.2 \pi, 0.6 \pi-1$ <br> ignore ref to point when $t=1$ can show on graph |
| 7 | (i) | Tension in string $T=m g \sin \alpha$ <br> For using $e=R \alpha-2 R / 3$ $1.8 \alpha-\sin \alpha-1.2=0$ <br> Finding $f(1.175)$ and $f(1.185)$ correctly correct conclusion | M1 <br> B1 <br> B1 <br> A1 <br> M1 <br> A1 <br> A1 <br> [7] | For using $T=\lambda x / L$ $m g \sin \alpha=1.2 m g\left(R a-\frac{2 R}{3}\right) \div \frac{2 R}{3}$ <br> AG establish result $\approx-0.008, \text { and } \approx+0.0065$ <br> $\mathrm{AG} \alpha=1.18$ correct to 3 significant figures | By iteration $\alpha=(1.2+\sin \alpha) / 1.8 \mathrm{M} 1$ <br> start [1, 2], and 1 iteration A1 at least 1 more iteration, and conclusion 1.18(0427) A1 |
| 7 | (ii) | Direction is towards $O$ | $\begin{aligned} & \text { B1 } \\ & \text { [1] } \end{aligned}$ |  |  |
| 7 | (iii) | Gain in $\mathrm{EE}=1.2 \mathrm{mg}(1.18 R-2 R / 3)^{2} \div(2 \mathrm{x} 2 R / 3)$ PE loss $=m g R(\cos 2 / 3-\cos 1.18)$ $\begin{aligned} & v^{2}= \\ & 2 g R\left[\cos 2 / 3-\cos 1.18-0.9(1.18-2 / 3)^{2}\right] \end{aligned}$ <br> Acceleration is $3.29 \mathrm{~ms}^{-2}$. | $\begin{gathered} \hline \text { M1* } \\ \text { A1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ \text { *M1 } \\ \text { A1 } \\ \text { [7] } \\ \hline \end{gathered}$ | For using $\mathrm{EE}=\lambda e^{2} \div(2 L)$ and $\mathrm{PE}=m g h$ ignore signs For using $1 / 2 m v^{2}=$ PE loss - EE gain <br> For using acceleration $=v^{2} / R$ | allow $\alpha$ for 1.18 for A1A1 allow sign errors <br> need 1.18 here If candidates use $m R \ddot{\theta}$ use equivalent scheme |

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